# Atomic data from the IRON Project 

# XXIV. Electron excitation of Li-like Fe XxIV between the $n=2$ and $n^{\prime}=2,3,4$ fine-structure levels 

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#### Abstract

Collision strengths for electron induced transitions in Li-like Fe xxiv are calculated using a BreitPauli R-matrix technique. The target has 15 fine structure states corresponding to $n \leq 4$. Autoionizing resonances are found to affect 18 of the 39 collision strengths, namely those for transitions $2 l j \rightarrow n^{\prime} l^{\prime} j^{\prime}$ where $n^{\prime}=2,3$. The present calculation is carried out only for electrons incident on the ground state with energies not exceeding 265 Ry . Beyond this we use the relativistic distorted wave collision strengths of Zhang et al. in order to complete our results. Thermally averaged collision strengths are computed and tabulated as functions of $\log T$ for transitions out of the $2 \mathrm{~s}_{1 / 2}, 2 \mathrm{p}_{1 / 2}$ and $2 \mathrm{p}_{3 / 2}$ states. The temperature range considered is $6.2 \leq \log T \leq 8.0$, Fe xxiv having its maximum coronal abundance when $\log T=7.2$.


Key words: Li-like iron - electron collision - atomic data

## 1. Introduction

The present calculation is part of an international collaboration known as the IRON Project (Hummer et al. 1993, referred to as Paper I) to obtain accurate collision rates for fine-structure transitions. Other papers in the series are given in the References section, while a complete list of IRON Project published papers and those in press is available at http://www.am.qub.ac.uk.

Collisionally induced transitions in Fe xxiv have been examined by several authors in the past, most of whom used distorted wave (DW) approximations with some allowance for relativistic effects. To our knowledge, Hayes (1979) is the only person who has done a close-coupling (CC) calculation for this ion. Her method, however, did not take account of exchange and furthermore she only applied it to one energy above the excitation thresholds of
her target in order to confirm that the coupling potentials could be neglected and that the DW approximation was sufficiently reliable. Hayes found that her CC and DW collision strengths at this particular energy agreed to within $5 \%$ for most transitions, except for $2 \mathrm{p}-3 \mathrm{~s}$ where the difference was about $20 \%$.

The results given by Hayes (1979), Mann (1983), Younger (1987) and Zhang et al. (1990) are from DW calculations, which are similar in so far as they all take account of relativistic effects but ignore channel coupling and therefore show no resonance structure. For this reason we confine our comparison to the most recent work, namely that of Zhang et al. (1990).

## 2. The calculation

Paper I gives the basic atomic theory, approximations and computer codes employed in the IRON Project. The CC approximation known as the R-matrix method is used. In the present case we have taken account of channel coupling up to the $n=4$ levels. Relativistic effects are allowed for as explained later.

The radial orbitals for the Li-like target are as follows: $P_{1 \mathrm{~s}}, P_{2 \mathrm{~s}}$ are from Clementi \& Roetti (1974). $P_{2 \mathrm{p}}$ is the 2 exponent function

$$
\begin{align*}
P_{2 \mathrm{p}}(r)= & 584.527 r^{2} \exp (-12.0960 r) \\
& +35.5799 r^{2} \exp (-27.4414 r) \tag{1}
\end{align*}
$$

which we obtained by using Hibbert's (1975) variational program CIV3 to minimise the energy of the $1 \mathrm{~s}^{2} 2 \mathrm{p}$ term. $P_{3 \mathrm{~s}}$ and $P_{3 \mathrm{~d}}$ are from Tully et al. (1990). Each remaining orbital $P_{n l}$, with $n l=3 \mathrm{p}, 4 \mathrm{~s}, 4 \mathrm{p}, 4 \mathrm{~d}, 4 \mathrm{f}$, has the minimum number of exponents dictated by $n l$. The values of these exponents were calculated using CIV3 to minimise the appropriate $1 \mathrm{~s}^{2} n l$ term energy. The orbital exponents for $n=3,4$ are given in Table 1 .

Table 1. Exponents for the $\mathrm{Fe}^{+23}$ radial orbitals with analytic form similar to that in (1). The coefficients are fixed by orthonormality conditions

| $P_{n l}$ | $r^{1}$ | $r^{2}$ | $r^{3}$ | $r^{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| 3s | 14.437 | 7.9574 | 8.0058 |  |
| 3p |  | 9.600263 | 7.847094 |  |
| 3d |  |  | 8.0050 |  |
| 4s | 13.360597 | 6.031843 | 6.055029 | 6.021776 |
| 4p |  | 8.209951 | 5.974792 | 5.957608 |
| 4d |  |  | 6.367924 | 5.971795 |
| 4f |  |  |  | 6.000065 |
|  |  |  |  |  |

Although configuration interaction (CI) wavefunctions are used to describe the target terms, in practice each term is dominated by a single configuration. The target energies used in the collision calculation were, with one important exception, adjusted to match the accurate experimental levels of Reader et al. (1992). These are given in Table 2 after being converted to Ry ( $1 \mathrm{Ry}=109737.32 \mathrm{~cm}^{-1}$ ). The exception concerns each pair of $4 l$ fine-structure levels which we forced to be degenerate with the corresponding $L S$ term.

Table 2. Fe xxiv: observed level energies $E_{i}$ (obs) in Ry from Reader et al. (1992). $\Delta=E_{i}(\mathrm{BP})-E_{i}(\mathrm{obs})$, where $E_{i}(\mathrm{BP})$ is from the Breit-Pauli R-matrix calculation

| $i$ | Term | $j$ | $E_{i}$ (obs) | uncertainty | $\Delta$ |
| :--- | :--- | :--- | :--- | :--- | ---: |
| 1 | $2 \mathrm{~s}^{2} \mathrm{~S}$ | $1 / 2$ | 0.0 |  |  |
| 2 | $2 \mathrm{p}{ }^{2} \mathrm{P}$ | $1 / 2$ | 3.57201 | 0.00007 | 0.004 |
| 3 |  | $3 / 2$ | 4.74549 | 0.00013 | -0.014 |
| 4 | $3 \mathrm{~s}{ }^{2} \mathrm{~S}$ | $1 / 2$ | 84.497 | 0.015 | 0.034 |
| 5 | $3 \mathrm{p}{ }^{2} \mathrm{P}$ | $1 / 2$ | 85.461 | 0.024 | 0.048 |
| 6 |  | $3 / 2$ | 85.815 | 0.025 | 0.024 |
| 7 | $3 \mathrm{~d}{ }^{2} \mathrm{D}$ | $3 / 2$ | 86.197 | 0.015 | 0.034 |
| 8 |  | $5 / 2$ | 86.321 | 0.022 | 0.019 |
| 9 | $4 \mathrm{~s}{ }^{2} \mathrm{~S}$ | $1 / 2$ | 113.584 | 0.006 | 0.035 |
| 10 | $4 \mathrm{p}{ }^{2} \mathrm{P}$ | $1 / 2$ | 113.990 | 0.015 | 0.030 |
| 11 |  | $3 / 2$ | 114.136 | 0.015 | 0.018 |
| 12 | $4 \mathrm{~d}^{2} \mathrm{D}$ | $3 / 2$ | 114.266 | 0.020 | 0.050 |
| 13 |  | $5 / 2$ | 114.321 | 0.025 | 0.040 |
| 14 | $4 \mathrm{f}^{2} \mathrm{~F}$ | $5 / 2$ | 114.342 | 0.015 | 0.028 |
| 15 |  | $7 / 2$ | 114.379 | 0.022 | 0.014 |

Theoretical $L S$-coupling oscillator strengths calculated using the present wavefunctions and observed transition energies (see Table 2) are compared in Table 3 with those which Peach et al. (1988) obtained in the Opacity Project. Our length (L) and velocity (V) forms agree well
and there is also fairly good overall agreement between our length oscillator strengths and those of Peach et al. (1988). This suggests that our choice of wavefunctions is satisfactory for computing reliable collision data.

Table 3. Oscillator strengths for $\mathrm{Fe}^{+23}$ calculated in the length ( L ) and velocity (V) gauge, compared with those from the Opacity Project (Peach et al. 1988)

| Transition | L | V | OP |
| :--- | :--- | :--- | :--- |
| $2 \mathrm{~s}-2 \mathrm{p}$ | 0.0525 | 0.0606 | 0.052 |
| $2 \mathrm{~s}-3 \mathrm{p}$ | 0.386 | 0.383 | 0.386 |
| $2 \mathrm{~s}-4 \mathrm{p}$ | 0.0956 | 0.0944 | 0.0955 |
| $3 \mathrm{~s}-3 \mathrm{p}$ | 0.0881 | 0.0947 | 0.0875 |
| $3 \mathrm{~s}-4 \mathrm{p}$ | 0.425 | 0.421 | 0.424 |
| $4 \mathrm{~s}-4 \mathrm{p}$ | 0.122 | 0.126 | 0.121 |
| $2 \mathrm{p}-3 \mathrm{~s}$ | 0.0170 | 0.0174 | 0.017 |
| $2 \mathrm{p}-4 \mathrm{~s}$ | 0.00365 | 0.00376 | 0.0037 |
| $3 \mathrm{p}-4 \mathrm{~s}$ | 0.0399 | 0.0395 | 0.0395 |
|  |  |  |  |
| $2 \mathrm{p}-3 \mathrm{~d}$ | 0.684 | 0.683 | 0.683 |
| $2 \mathrm{p}-4 \mathrm{~d}$ | 0.122 | 0.122 | 0.157 |
| $3 \mathrm{p}-3 \mathrm{~d}$ | 0.0149 | 0.0171 | 0.0151 |
| $3 \mathrm{p}-4 \mathrm{~d}$ | 0.599 | 0.597 | 0.598 |
| $4 \mathrm{p}-3 \mathrm{~d}$ | 0.0123 | 0.0124 | 0.0123 |
| $4 \mathrm{p}-4 \mathrm{~d}$ | 0.0263 | 0.0274 | 0.0268 |
|  |  |  |  |
| $3 \mathrm{~d}-4 \mathrm{f}$ | 1.016 | 1.016 |  |
| $4 \mathrm{~d}-4 \mathrm{f}$ | 0.00076 | 0.00078 |  |

Collision strengths for fine structure transitions are obtained from two R-matrix calculations which we now describe.
$\mathbf{B P}$ is a 15 -level fine-structure calculation using the BreitPauli Hamiltonian version of the R-matrix program (Berrington et al. 1995). This is the most accurate approach when the collision energy is comparable to the level splitting in the target. This method was used in the scattering energy range up to the highest threshold.
JAJOM is a 9 -term $L S$-coupling calculation plus a transformation to intermediate coupling which we apply to the reactance matrix using the JAJOM program written by Saraph (1978). This was used from just above the highest threshold up to a scattering energy of 265 Ry. For allowed transitions we use a "top-up" procedure which consists in assuming that beyond $J=80$ the partial collision strengths form a geometrical series. In this way we are able to complete the sum to infinity of partial waves analytically.
For the purpose of comparing our results numerically with those of Zhang et al. (1990) we choose an energy that lies above the highest target term included in our calculation, namely 4 f . The comparison is shown in Table 4
where it can be seen that differences are less than $10 \%$ for all transitions out of $2 \mathrm{~s}_{1 / 2}$. For transitions from the 2 p levels the differences are greater with some of them as high as $14 \%$.

Table 4. A comparison of R-matrix (BP) and distorted-wave (DW) collision strengths for excitation of $\mathrm{Fe}^{+23}$ at 132.25 Ry . The DW results are from Zhang et al. (1990)

| Transition | BP | DW |
| ---: | :--- | :--- |
| $2 \mathrm{~s}_{1 / 2}-2 \mathrm{p}_{1 / 2}$ | 0.254 | 0.253 |
| $-2 \mathrm{p}_{3 / 2}$ | 0.472 | 0.475 |
| $-3 \mathrm{~s}_{1 / 2}$ | 0.0137 | 0.0151 |
| $-3 \mathrm{p}_{1 / 2}$ | 0.0053 | 0.0054 |
| $-3 \mathrm{p}_{3 / 2}$ | 0.0101 | 0.0099 |
| $-3 \mathrm{~d}_{3 / 2}$ | 0.0118 | 0.0118 |
| $-3 \mathrm{~d}_{5 / 2}$ | 0.0176 | 0.0177 |
| $-4 \mathrm{~s}_{1 / 2}$ | 0.0025 | 0.0029 |
| $-4 \mathrm{p}_{1 / 2}$ | 0.0011 | 0.0010 |
| $-4 \mathrm{p}_{3 / 2}$ | 0.0022 | 0.0020 |
| $-4 \mathrm{~d}_{3 / 2}$ | 0.0021 | 0.0020 |
| $-4 \mathrm{~d}_{5 / 2}$ | 0.0031 | 0.0030 |
| $-4 \mathrm{f}_{5 / 2}$ | 0.0012 | 0.0011 |
| $-4 \mathrm{f}_{7 / 2}$ | 0.0016 | 0.0014 |
| $2 \mathrm{p}_{1 / 2}-2 \mathrm{p}_{3 / 2}$ | 0.0196 | 0.0191 |
| $-3 \mathrm{~s}_{1 / 2}$ | 0.0011 | 0.0010 |
| $-3 \mathrm{p}_{1 / 2}$ | 0.0142 | 0.0155 |
| $-3 \mathrm{p}_{3 / 2}$ | 0.0034 | 0.0034 |
| $-3 \mathrm{~d}_{3 / 2}$ | 0.0609 | 0.0613 |
| $-3 \mathrm{~d}_{5 / 2}$ | 0.0065 | 0.0056 |

The approximations in each method are comparable at higher energies and any differences here are presumably caused by different "topping" up procedures and possibly by the fact that Zhang et al. (1990) use orbitals obtained by solving the Dirac equation with a Dirac-Fock-Slater potential. We expect really important differences to occur only at lower energies where resonance structures such as those shown in Figs. 1-2 and 4-7 occur. Resonances can have a big effect on effective collision strengths, as seen in Figs. 8 and 9.

Thermal averaging of the collision strengths is done using the "linear interpolation" method described by Burgess \& Tully (1992). The resulting effective collision strengths $\Upsilon$ are given in Table 5 for the astrophysically important temperature range $6.2 \leq \log T \leq 8.0$ where $\mathrm{Fe}^{+23}$ is abundant under conditions of coronal ionization equilibrium (see Arnaud \& Rothenflug 1985). For temperatures below two million degrees the abundance will be negligible. For this reason we begin our tabulation of $\Upsilon$ in Table 5 at $\log T=6.2$. Astrophysical situations may exist where $\mathrm{Fe}^{+23}$ is abundant at temperatures lower than this; in these cases one would need to extend the temperature range below $10^{6.2} \mathrm{~K}$. This should pose no problem since our collision strengths will be preserved for pos-
terity at the CDS (Centre de données astronomiques de Strasbourg) and some other databanks.


Final electron energy in Ry
Fig. 1. $\mathrm{Fe}^{+23}\left(2 \mathrm{~s}_{1 / 2} \rightarrow 2 \mathrm{p}_{1 / 2}\right)$ collision strength shown over the range $0 \leq E_{f} \leq 110.80699$ (i.e. from $2 p_{1 / 2}$ to $4 \mathrm{f}_{7 / 2}$ ). Full line: present Breit-Pauli calculation; broken line: DW calculation by Zhang et al. (1990)

## 3. Discussion of resonances and their effect on $\Upsilon$

Of our energy dependent collision strengths only those for the 18 transitions between $n=2$ and $n^{\prime}=2,3$ are perturbed by autoionising resonances. In some cases the resonances greatly increase the effective collision strengths. But in general this happens at temperatures below a million degrees so it is not of particular astrophysical significance since, as mentioned earlier, under conditions of coronal ionization equilibrium the abundance of $\mathrm{Fe}^{+23}$ is essentially zero at such "low" temperatures.

It would be tedious to discuss in detail the energy dependence of each of the 39 collision strengths we have calculated. Here we select a few cases which serve to illustrate the main features. For conciseness and convenience we label the target energy levels $n l_{j}$ in ascending order with the index shown in Table 2. This starts at 1 for the ground level $\left(2 \mathrm{~s}_{1 / 2}\right)$ and ends at 15 for the highest level $\left(4 f_{7 / 2}\right)$.

Transitions $1 \rightarrow 2$ and $1 \rightarrow 3$ are optically allowed and, apart from their numerical values, the collision strengths $\Omega_{12}$ and $\Omega_{13}$ have almost identical resonance patterns. Figure 1 shows $\Omega_{12}$ from threshold up to the energy of the $4 \mathrm{f}_{7 / 2}$ level. Note that we plot $\Omega$ against the final electron energy $E_{f}$ in Rydberg units. The initial energy of the colliding electron $E_{i}$ relative to the the lower state is

Table 5. $\mathrm{Fe}^{+23}$ effective collision strengths for $6.2 \leq \log T \leq 8.0$

| Transition | 6.2 | 6.4 | 6.6 | 6.8 | 7.0 | 7.2 | 7.4 | 7.6 | 7.8 | 8.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2 \mathrm{~s}_{1 / 2}-2 \mathrm{p}_{1 / 2}$ | $1.661^{-1}$ | $1.747^{-1}$ | $1.853^{-1}$ | $1.976{ }^{-1}$ | $2.117^{-1}$ | $2.278^{-1}$ | $2.459^{-1}$ | $2.659^{-1}$ | $2.873^{-1}$ | $3.095^{-1}$ |
| $2 \mathrm{~s}_{1 / 2}-2 \mathrm{p}_{3 / 2}$ | $3.172^{-1}$ | $3.331^{-1}$ | $3.523^{-1}$ | $3.744^{-1}$ | $4.000^{-1}$ | $4.298^{-1}$ | $4.642^{-1}$ | $5.027^{-1}$ | $5.445^{-1}$ | $5.884^{-1}$ |
| $2 \mathrm{~s}_{1 / 2}-3 \mathrm{~s}_{1 / 2}$ | $1.603^{-2}$ | $1.562^{-2}$ | $1.522^{-2}$ | $1.493{ }^{-2}$ | $1.484^{-2}$ | $1.499^{-2}$ | $1.534^{-2}$ | $1.581{ }^{-2}$ | $1.631^{-2}$ | $1.677^{-2}$ |
| $2 \mathrm{~s}_{1 / 2}-3 \mathrm{p}_{1 / 2}$ | $5.573^{-3}$ | $5.533^{-3}$ | $5.566^{-3}$ | $5.792^{-3}$ | $6.328^{-3}$ | $7.284^{-3}$ | $8.747^{-3}$ | $1.077^{-2}$ | $1.339^{-2}$ | $1.658^{-2}$ |
| $2 \mathrm{~s}_{1 / 2}-3 \mathrm{p}_{3 / 2}$ | $1.038^{-2}$ | $1.041^{-2}$ | $1.052^{-2}$ | $1.097^{-2}$ | $1.198^{-2}$ | $1.378^{-2}$ | $1.650^{-2}$ | $2.027^{-2}$ | $2.511^{-2}$ | $3.101^{-2}$ |
| $2 \mathrm{~s}_{1 / 2}-3 \mathrm{~d}_{3 / 2}$ | $1.280^{-2}$ | $1.274^{-2}$ | $1.263{ }^{-2}$ | $1.262^{-2}$ | $1.284^{-2}$ | $1.333^{-2}$ | $1.408^{-2}$ | $1.503^{-2}$ | $1.608^{-2}$ | $1.714^{-2}$ |
| $2 \mathrm{~s}_{1 / 2}-3 \mathrm{~d}_{5 / 2}$ | $1.960^{-2}$ | $1.954^{-2}$ | $1.930^{-2}$ | $1.920^{-2}$ | $1.943^{-2}$ | $2.009^{-2}$ | $2.118^{-2}$ | $2.264^{-2}$ | $2.444^{-2}$ | $2.644^{-2}$ |
| $2 \mathrm{~s}_{1 / 2}-4 \mathrm{~s}_{1 / 2}$ | $2.497{ }^{-3}$ | $2.507^{-3}$ | $2.526^{-3}$ | $2.559^{-3}$ | $2.621^{-3}$ | $2.719^{-3}$ | $2.842^{-3}$ | $2.976^{-3}$ | $3.105^{-3}$ | $3.224^{-3}$ |
| $2 \mathrm{~s}_{1 / 2}-4 \mathrm{p}_{1 / 2}$ | $1.100^{-3}$ | $1.128^{-3}$ | $1.174^{-3}$ | $1.251^{-3}$ | $1.374^{-3}$ | $1.562^{-3}$ | $1.834^{-3}$ | $2.204^{-3}$ | $2.681^{-3}$ | $3.262^{-3}$ |
| $2 \mathrm{~s}_{1 / 2}-4 \mathrm{p}_{3 / 2}$ | $2.176^{-3}$ | $2.231^{-3}$ | $2.323^{-3}$ | $2.472^{-3}$ | $2.705^{-3}$ | $3.056^{-3}$ | $3.564^{-3}$ | $4.267^{-3}$ | $5.184^{-3}$ | $6.315^{-3}$ |
| $2 \mathrm{~s}_{1 / 2}-4 \mathrm{~d}_{3 / 2}$ | $2.095^{-3}$ | $2.090^{-3}$ | $2.086^{-3}$ | $2.086^{-3}$ | $2.098^{-3}$ | $2.129^{-3}$ | $2.188^{-3}$ | $2.277^{-3}$ | $2.397^{-3}$ | $2.537^{-3}$ |
| $2 \mathrm{~s}_{1 / 2}-4 \mathrm{~d}_{5 / 2}$ | $3.168^{-3}$ | $3.160^{-3}$ | $3.154^{-3}$ | $3.158^{-3}$ | $3.177^{-3}$ | $3.224^{-3}$ | $3.311^{-3}$ | $3.438^{-3}$ | $3.598^{-3}$ | $3.776^{-3}$ |
| $2 \mathrm{~s}_{1 / 2}-4 \mathrm{f}_{5 / 2}$ | $1.233^{-3}$ | $1.212^{-3}$ | $1.189^{-3}$ | $1.166^{-3}$ | $1.146^{-3}$ | $1.133^{-3}$ | $1.130^{-3}$ | $1.136^{-3}$ | $1.148^{-3}$ | $1.164^{-3}$ |
| $2 \mathrm{~s}_{1 / 2}-4 \mathrm{f}_{7 / 2}$ | $1.646^{-3}$ | $1.618^{-3}$ | $1.587^{-3}$ | $1.556^{-3}$ | $1.529^{-3}$ | $1.511^{-3}$ | $1.506^{-3}$ | $1.515^{-3}$ | $1.538^{-3}$ | $1.568^{-3}$ |
| $2 \mathrm{p}_{1 / 2}-2 \mathrm{p}_{3 / 2}$ | $3.734^{-2}$ | $4.120^{-2}$ | $4.291^{-2}$ | $4.166^{-2}$ | $3.807^{-2}$ | $3.346^{-2}$ | $2.896^{-2}$ | $2.516^{-2}$ | $2.225^{-2}$ | $2.017^{-2}$ |
| $2 \mathrm{p}_{1 / 2}-3 \mathrm{~s}_{1 / 2}$ | $4.738^{-3}$ | $3.722^{-3}$ | $2.950^{-3}$ | $2.383^{-3}$ | $2.004^{-3}$ | $1.796^{-3}$ | $1.748^{-3}$ | $1.846^{-3}$ | $2.076^{-3}$ | $2.420^{-3}$ |
| $2 \mathrm{p}_{1 / 2}-3 \mathrm{p}_{1 / 2}$ | $1.650^{-2}$ | $1.604^{-2}$ | $1.562^{-2}$ | $1.529^{-2}$ | $1.513^{-2}$ | $1.517^{-2}$ | $1.538^{-2}$ | $1.569^{-2}$ | $1.602^{-2}$ | $1.634^{-2}$ |
| $2 \mathrm{p}_{1 / 2}-3 \mathrm{p}_{3 / 2}$ | $7.156^{-3}$ | $6.702^{-3}$ | $5.999^{-3}$ | $5.245^{-3}$ | $4.568^{-3}$ | $4.024^{-3}$ | $3.626^{-3}$ | $3.363^{-3}$ | $3.214^{-3}$ | $3.154^{-3}$ |
| $2 \mathrm{p}_{1 / 2}-3 \mathrm{~d}_{3 / 2}$ | $5.389^{-2}$ | $5.534^{-2}$ | $5.734^{-2}$ | $6.046^{-2}$ | $6.524^{-2}$ | $7.223^{-2}$ | $8.188^{-2}$ | $9.456^{-2}$ | $1.104^{-1}$ | $1.293^{-1}$ |
| $2 \mathrm{p}_{1 / 2}-3 \mathrm{~d}_{5 / 2}$ | $1.267^{-2}$ | $1.189^{-2}$ | $1.063{ }^{-2}$ | $9.202^{-3}$ | $7.834^{-3}$ | $6.629^{-3}$ | $5.633^{-3}$ | $4.859^{-3}$ | $4.296^{-3}$ | $3.913^{-3}$ |
| $2 \mathrm{p}_{1 / 2}-4 \mathrm{~s}_{1 / 2}$ | $4.298^{-4}$ | $4.128^{-4}$ | $3.933^{-4}$ | $3.730^{-4}$ | $3.549^{-4}$ | $3.437^{-4}$ | $3.448^{-4}$ | $3.626^{-4}$ | $3.989^{-4}$ | $4.533^{-4}$ |
| $2 \mathrm{p}_{1 / 2}-4 \mathrm{p}_{1 / 2}$ | $2.745^{-3}$ | $2.734^{-3}$ | $2.723^{-3}$ | $2.721^{-3}$ | $2.741^{-3}$ | $2.794^{-3}$ | $2.873^{-3}$ | $2.958^{-3}$ | $3.038^{-3}$ | $3.104^{-3}$ |
| $2 \mathrm{p}_{1 / 2}-4 \mathrm{p}_{3 / 2}$ | $1.402^{-3}$ | $1.343^{-3}$ | $1.268^{-3}$ | $1.179^{-3}$ | $1.081^{-3}$ | $9.818^{-4}$ | $8.916^{-4}$ | $8.177^{-4}$ | $7.634^{-4}$ | $7.283^{-4}$ |
| $2 \mathrm{p}_{1 / 2}-4 \mathrm{~d}_{3 / 2}$ | $9.901^{-3}$ | $1.011^{-2}$ | $1.043^{-2}$ | $1.091^{-2}$ | $1.161^{-2}$ | $1.261^{-2}$ | $1.396^{-2}$ | $1.573^{-2}$ | $1.793^{-2}$ | $2.054^{-2}$ |
| $2 \mathrm{p}_{1 / 2}-4 \mathrm{~d}_{5 / 2}$ | $2.571^{-3}$ | $2.432^{-3}$ | $2.253^{-3}$ | $2.035^{-3}$ | $1.790^{-3}$ | $1.534^{-3}$ | $1.290^{-3}$ | $1.070^{-3}$ | $8.847^{-4}$ | $7.351^{-4}$ |
| $2 \mathrm{p}_{1 / 2}-4 \mathrm{f}_{5 / 2}$ | $1.698^{-3}$ | $1.711^{-3}$ | $1.742^{-3}$ | $1.802^{-3}$ | $1.900^{-3}$ | $2.046^{-3}$ | $2.245^{-3}$ | $2.492^{-3}$ | $2.772^{-3}$ | $3.063^{-3}$ |
| $2 \mathrm{p}_{1 / 2}-4 \mathrm{f}_{7 / 2}$ | $1.051^{-3}$ | $9.836^{-4}$ | $8.993^{-4}$ | $8.017^{-4}$ | $6.971^{-4}$ | $5.947^{-4}$ | $5.035^{-4}$ | $4.299^{-4}$ | $3.760^{-4}$ | $3.402^{-4}$ |
| $2 \mathrm{p}_{3 / 2}-3 \mathrm{~s}_{1 / 2}$ | $1.044^{-2}$ | $8.312^{-3}$ | $6.629^{-3}$ | $5.355^{-3}$ | $4.483^{-3}$ | $3.996^{-3}$ | $3.872^{-3}$ | $4.084^{-3}$ | $4.604^{-3}$ | $5.392^{-3}$ |
| $2 \mathrm{p}_{3 / 2}-3 \mathrm{p}_{1 / 2}$ | $8.007^{-3}$ | $7.342^{-3}$ | $6.490^{-3}$ | $5.632^{-3}$ | $4.886^{-3}$ | $4.303^{-3}$ | $3.888^{-3}$ | $3.624^{-3}$ | $3.487^{-3}$ | $3.444^{-3}$ |
| $2 \mathrm{p}_{3 / 2}-3 \mathrm{p}_{3 / 2}$ | $4.098^{-2}$ | $4.002^{-2}$ | $3.863^{-2}$ | $3.726^{-2}$ | $3.624^{-2}$ | $3.568^{-2}$ | $3.556^{-2}$ | $3.577^{-2}$ | $3.616^{-2}$ | $3.664^{-2}$ |
| $2 \mathrm{p}_{3 / 2}-3 \mathrm{~d}_{3 / 2}$ | $2.691^{-2}$ | $2.621^{-2}$ | $2.503^{-2}$ | $2.390^{-2}$ | $2.319^{-2}$ | $2.314^{-2}$ | $2.389^{-2}$ | $2.552^{-2}$ | $2.803^{-2}$ | $3.134^{-2}$ |
| $2 \mathrm{p}_{3 / 2}-3 \mathrm{~d}_{5 / 2}$ | $1.108^{-1}$ | $1.129^{-1}$ | $1.156^{-1}$ | $1.202^{-1}$ | $1.279^{-1}$ | $1.399^{-1}$ | $1.569^{-1}$ | $1.795^{-1}$ | $2.080^{-1}$ | $2.419^{-1}$ |
| $2 \mathrm{p}_{3 / 2}-4 \mathrm{~s}_{1 / 2}$ | $8.680^{-4}$ | $8.342^{-4}$ | $7.957^{-4}$ | $7.566^{-4}$ | $7.232^{-4}$ | $7.053^{-4}$ | $7.139^{-4}$ | $7.576^{-4}$ | $8.400^{-4}$ | $9.600^{-4}$ |
| $2 \mathrm{p}_{3 / 2}-4 \mathrm{p}_{1 / 2}$ | $1.376^{-3}$ | $1.319^{-3}$ | $1.246^{-3}$ | $1.161^{-3}$ | $1.068^{-3}$ | $9.774^{-4}$ | $8.972^{-4}$ | $8.360{ }^{-4}$ | $7.998^{-4}$ | $7.902^{-4}$ |
| $2 \mathrm{p}_{3 / 2}-4 \mathrm{p}_{3 / 2}$ | $7.294^{-3}$ | $7.216^{-3}$ | $7.123^{-3}$ | $7.040^{-3}$ | $7.016^{-3}$ | $7.078^{-3}$ | $7.204^{-3}$ | $7.349^{-3}$ | $7.479^{-3}$ | $7.580^{-3}$ |
| $2 \mathrm{p}_{3 / 2}-4 \mathrm{~d}_{3 / 2}$ | $5.102^{-3}$ | $4.974^{-3}$ | $4.820^{-3}$ | $4.653^{-3}$ | $4.499^{-3}$ | $4.397^{-3}$ | $4.387^{-3}$ | $4.503^{-3}$ | $4.759^{-3}$ | $5.152^{-3}$ |
| $2 \mathrm{p}_{3 / 2}-4 \mathrm{~d}_{5 / 2}$ | $2.029^{-2}$ | $2.056^{-2}$ | $2.099^{-2}$ | $2.169^{-2}$ | $2.278^{-2}$ | $2.440^{-2}$ | $2.671^{-2}$ | $2.981^{-2}$ | $3.374^{-2}$ | $3.846^{-2}$ |
| $2 \mathrm{p}_{3 / 2}-4 \mathrm{f}_{5 / 2}$ | $1.908^{-3}$ | $1.823^{-3}$ | $1.721^{-3}$ | $1.610^{-3}$ | $1.502^{-3}$ | $1.411^{-3}$ | $1.350^{-3}$ | $1.326^{-3}$ | $1.338^{-3}$ | $1.376^{-3}$ |
| $2 \mathrm{p}_{3 / 2}-4 \mathrm{f}_{7 / 2}$ | $3.834^{-3}$ | $3.812^{-3}$ | $3.809^{-3}$ | $3.850^{-3}$ | $3.954^{-3}$ | $4.142^{-3}$ | $4.432{ }^{-3}$ | $4.821^{-3}$ | $5.285^{-3}$ | $5.787^{-3}$ |

given by $E_{i}=E_{f}+E_{i f}$, where $E_{i f}$ is the transition energy and can be obtained from Table 2. At $E_{f} \simeq 0$ there is an indication in Fig. 1 of some structure in the collision strength. Suspecting that this is caused by resonances converging on the level $2 \mathrm{p}_{3 / 2}$, we give a blow-up in Fig. 2 of the $2 \mathrm{p}_{1 / 2}-2 \mathrm{p}_{3 / 2}$ energy interval $\left(0 \leq E_{f} \leq 1.17348\right)$. Here we clearly see a number of narrow, well separated resonances. A striking amount of detail, which is completely hidden in Fig. 1, is now revealed. The apparently
random way the peaks of these small resonances vary is caused by not using a sufficiently small steplength in energy for the purpose of scanning. To do so would greatly increase the amount of computing time required and make the whole calculation extremely arduous. The reason why resonances occur over less than half the interval is because the search for them was stopped after a finite number had been located. Figure 3 shows $\Upsilon_{12}$ as a function of temperature over the interval $4 \leq \log T \leq 8$. One can see a slight


Fig. 2. $\mathrm{Fe}^{+23}\left(2 \mathrm{~s}_{1 / 2} \rightarrow 2 \mathrm{p}_{1 / 2}\right)$ collision strength shown over the range $0 \leq E_{f} \leq 1.17348$ (i.e. from $2 \mathrm{p}_{1 / 2}$ to $2 \mathrm{p}_{3 / 2}$ ). Full line: present Breit-Pauli calculation; broken line: DW calculation by Zhang et al. (1990)
increase in $\Upsilon$ for temperatures below about $10^{6}$. This is due to the resonances shown in Fig. 2; those at higher energies have a much smaller effect on $\Upsilon$ and produce a barely perceptible increase at temperatures above $10^{6}$.

The optically forbidden transition $1 \rightarrow 4$ is a much more interesting case. Here we cover the range from $E_{f}=$ 0 to the $4 f_{7 / 2}$ level by means of four separate plots in order to illustrate the varied structure of the resonances. Figure 4 covers the range $0 \leq E_{f} \leq 0.96383$, which corresponds to the interval $3 \mathrm{~s}_{1 / 2}-3 \mathrm{p}_{1 / 2}$. A thick forest of resonances is seen to occur here. A comparable forest also occurs in Fig. 5 for the range $0.96383 \leq E_{f} \leq 1.31783$ (interval $3 \mathrm{p}_{1 / 2}-3 \mathrm{p}_{3 / 2}$ ), while in Fig. 6 for the range $1.31783 \leq E_{f} \leq 1.69983$ (interval $3 \mathrm{p}_{3 / 2}-3 \mathrm{~d}_{3 / 2}$ ), the forest of resonances is preceded by a collection of isolated peaks. Between the levels $3 \mathrm{~d}_{3 / 2}$ and $3 \mathrm{~d}_{5 / 2}$ there are no resonances and no graph is shown. In Fig. 7, which covers the interval $3 \mathrm{~d}_{5 / 2}-4 \mathrm{~s}_{1 / 2}\left(1.82383 \leq E_{f} \leq 29.08683\right)$, a striking series of Rydberg resonances appears. The process of delineation was stopped a short way from the $4 \mathrm{~s}_{1 / 2}$ level after 5 groups of resonances had been delineated. Figure 8 shows that after thermal averaging the effect on $\Upsilon_{14}$ of all those resonances is considerable at temperatures below about one million degrees.

Finally, the transition $2 \rightarrow 3$, namely $2 \mathrm{p}_{1 / 2}-2 \mathrm{p}_{3 / 2}$, is the only case encountered where resonances have a really big effect ( $50 \%$ increase above the background) at temperatures near ten million degrees, see Fig. 9.

The dotted curves shown in all graphs of $\Omega$ or $\Upsilon$ represent the DW calculations by Zhang et al. (1990). In order


Fig. 3. $\mathrm{Fe}^{+23}\left(2 \mathrm{~s}_{1 / 2} \rightarrow 2 \mathrm{p}_{1 / 2}\right)$ effective collision strength for the temperature range $4 \leq \log T \leq 8$. Full line: IRON Project; broken line: Zhang et al. (1990)
to make use of their tabulated collision strengths we first fitted them by cubic splines using the program OMEUPS (Burgess \& Tully 1992). Each spline fit was then used to generate a tableau of collision strength values for the purpose of plotting. The same tableau was used to compute a thermally averaged collision strength in order to make a graphical comparison with the present IRON Project results.

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Final electron energy in Ry

Fig. 4. $\mathrm{Fe}^{+23}\left(2 \mathrm{~s}_{1 / 2} \rightarrow 3 \mathrm{~s}_{1 / 2}\right)$ collision strength shown over the range $0 \leq E_{f} \leq 0.96383$ (i.e. from $3 \mathrm{~s}_{1 / 2}$ to $3 \mathrm{p}_{3 / 2}$ ). Full line: present Breit-Pauli calculation; broken line: DW calculation by Zhang et al. (1990)


Fig. 5. $\mathrm{Fe}^{+23}\left(2 \mathrm{~s}_{1 / 2} \rightarrow 3 \mathrm{~s}_{1 / 2}\right)$ collision strength shown over the range $0.96383 \leq E_{f} \leq 1.31783$ (i.e. from $3 \mathrm{p}_{1 / 2}$ to $3 \mathrm{p}_{3 / 2}$ ). Full line: present Breit-Pauli calculation; broken line: DW calculation by Zhang et al. (1990)


Final electron energy in Ry

Fig. 6. $\mathrm{Fe}^{+23}\left(2 \mathrm{~s}_{1 / 2} \rightarrow 3 \mathrm{~s}_{1 / 2}\right)$ collision strength shown over the range $1.31783 \leq E_{f} \leq 1.69983$ (i.e. from $3 \mathrm{p}_{3 / 2}$ to $3 \mathrm{~d}_{3 / 2}$ ). Full line: present Breit-Pauli calculation; broken line: DW calculation by Zhang et al. (1990)


Fig. 7. $\mathrm{Fe}^{+23}\left(2 \mathrm{~s}_{1 / 2} \rightarrow 3 \mathrm{~s}_{1 / 2}\right)$ collision strength shown over the range $1.82383 \leq E_{f} \leq 29.08683$ (i.e. from $3 d_{5 / 2}$ to $4 s_{1 / 2}$ ). Full line: present Breit-Pauli calculation; broken line: DW calculation by Zhang et al. (1990)


Fig. 8. $\mathrm{Fe}^{+23}\left(2 \mathrm{~s}_{1 / 2} \rightarrow 3 \mathrm{~s}_{1 / 2}\right)$ effective collision strength for the temperature range $4 \leq \log T \leq 8$. Full line: IRON Project; broken line: Zhang et al. (1990)

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Fig. 9. $\mathrm{Fe}^{+23}\left(2 \mathrm{p}_{1 / 2} \rightarrow 2 \mathrm{p}_{3 / 2}\right)$ effective collision strength for the temperature range $4 \leq \log T \leq 8$. Full line: IRON Project; broken line: Zhang et al. (1990)

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